Can one predict upper level winds in midlatitudes without knowledge of eddy mean flow interactions?  
A thermodynamic view of baroclinic waves

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Motivation: the study by Green et al. (1966)

Strength of jetstream well predicted from application of Bernoulli’s equation:

$$v_c^2/2 + gz + c_p T + l_v q = cst$$

Schematic of the isentropic flow relative to a travelling wave
Objectives

• Test the existence of a link between low level heat content / SST and jetstream strength.

• If there is, assess the oceanic region of maximum influence.

• Do all this without studying eddy-mean flow interactions (!)
Part 1. A minimal model of baroclinic waves
“Sloping convection”:

\[ \Delta KE \approx \frac{1}{2} N^2 \Delta z^2 \]

\[ \Delta PE = g \Delta z \]

Eric Eady
(Imperial College!)
Thermodynamic view: \[ \Delta KE + \Delta PE + \Delta h \approx 0 \]

(conservation of energy by a Lagrangian parcel)

\[ \Delta PE = g \Delta z \]

NB: \[ h = c_p T + l_v q \]
The thermodynamic constraint: \( \Delta h \approx \Delta h(SST) \)

- In a reversible moist adiabatic ascent from the top of the boundary layer (RH\(\sim\)cst), the loss of enthalpy \( \Delta h \) is a simple function of SST.

Example of calculation between 800hPa & 300hPa
Combining dynamics & thermodynamics

• Consider energy conversion during adiabatic ascent,

\[ \Delta KE + \Delta PE = -\Delta h = |\Delta h| \]

• And the previous equation,

\[ \Delta KE = \frac{1}{2} N^2 \Delta z^2 - \Delta KE_o \]

• With:

\[ \Delta PE = g \Delta z \]
Combining dynamics & thermodynamics

• Solving the 2\textsuperscript{nd} order polynomial:

\[
\Delta PE = \left(\frac{g}{N}\right)^2 \left(\sqrt{1 + \frac{2(|\Delta h| + \Delta KE_o)N^2}{g^2}} - 1\right)
\]

\[
\Delta KE = -\Delta KE_o + \frac{1}{2} \left(\frac{g}{N}\right)^2 \left(\sqrt{1 + \frac{2(|\Delta h| + \Delta KE_o)N^2}{g^2}} - 1\right)^2
\]

• Small parameter:

\[
\varepsilon \equiv (|\Delta h| + \Delta KE_o)/(g/N)^2 \approx |\Delta h|/(g/N)^2 \ll 1
\]
Part 2. Testing the predictions against “observations”

- ERA interim data
- Daily (12.00 UTC)
- 1980-2012 (December through February)
Methodology

- No calculation of trajectories
- Impose conservation of energy between A(t-1day) and B(t)
- Only consider cases where KE is max at B(t) and $\omega<0$ at A(t-1day)
Number of KEmax events within $5^\circ \times 5^\circ$ boxes per season

→ Rare events at a given location, but always present somewhere on a given day
An example of snapshot at 300hPa
(KE in black CI=1kJ/kg, relative humidity in color)
Relationship between KE\textsubscript{max} and upper level relative humidity

- Expect moist air on the equatorward flank of the jet (below jet core)
- Expect a mix of dry and moist air at the core of the jet

NB: streamlines in (y,p) coordinates so representative of mass transport

Eliassen (1962)
PDFs for KE max events at 300hPa
(ERAinterim, DJF 1980-2012)

• About 40% of the events have RH > 0.6

• The distribution peaks near KE = 3kJ/kg (U~77m/s).

• Suggestion that moist profiles (RH > 0.6) have stronger low level ascent and broader distribution of KE (not shown here).
Test of the model:

\[ \Delta KE(\text{obs}) = \Delta KE_{\text{ideal}} - \Delta KE_0 \]

- Model is in best agreement with observations for KEmax events with relative humidity \( RH > 2/3 \)

NB: results shown are for winter averages. Daily calculation for \( RH > 2/3 \) has a slope \( \sim 0.7 \)
KEmax events with RH>2/3

- Number of events per winter within 5 X 5 degree boxes

- Number of events with the right energy at low level *one day earlier* (again within 5X5 degree boxes)
Link between SST and Δh for KEmax events with RH>2/3

- As SST increases, |Δh| increases

- However, the buffer effect of moist adiabats limits the sensitivity of Δh to SST (slope~3kJ/kg per 6K)

\[ R^2 = 0.72 \]

NB: for this and subsequent scatterplots, one cross for each winter (averaging of daily calculations)
Test of the model: actual scatter plots for KE_{max} events with RH>2/3

\[ \Delta KE \approx \frac{1}{2} \left( \frac{N}{g} \right)^2 |\Delta h|^2 - \Delta KE_o \]

- Good agreement considering the minimal GFD information

\[ R^2 = 0.4 \]

\[ R^2 = 0.41 \]
Test of the model: actual scatter plots for KE_{max} events with RH>2/3

- winter-to-winter variability in ΔKE is driven by changes in N^2, not by changes in Δh.

\[
\Delta KE \approx \frac{1}{2} \left( \frac{N}{g} \right)^2 |\Delta h|^2 - \Delta KE_o
\]
Summary

• There is skill in predicting jets from simple energetic arguments:

\[ KE \sim (\Delta h)^2 N^2, \quad \Delta h = \Delta h(SST) \]


• For North Atlantic “moist” jets, the most likely source of ascent coincides with the “warm tongue” of the Gulf Stream at subtropical latitudes.

• The SST impact on \( \Delta h \) is mostly through moisture (destabilizing) and is buffered by an enhancement of the moist stratification (stabilizing)

• As a result, the variability of winter-to-winter upper level KE is dominated by changes in stratification.
Implications for North Atlantic ocean-atmosphere interactions

- The thermodynamic control $\Delta h(SST)$ is qualitatively relevant to the positive feedback between the NAO/SST tripole interaction, but likely to be weak.
- The real “leverage” is on the oceanic control of $N^2$.

Sheldon et al. (2014)
extras
Implication of the model for a detection of an oceanic influence on the jet stream

- The dependence of KE on low level heat content is multiplicative:

$$\Delta KE \sim (\Delta h)^2 N^2$$

$$\Delta PE \sim \Delta h$$

which might be the fundamental reason why it is so difficult to extract an oceanic forcing in the extra-tropics.

Statistical model: 500 ensemble members
Implication of the model for a detection of an oceanic influence on the jet stream

• The dependence of KE on low level heat content is multiplicative:

\[ \Delta KE \sim (\Delta h)^2 N^2 \]

\[ \Delta PE \sim \Delta h \]

which might be the fundamental reason why it is so difficult to extract an oceanic forcing in the extra-tropics.

Statistical model: 50 ensemble members
Test of the model: actual scatter plots for a threshold RH = 2/3

- Yr-to-yr variability in ΔKE is driven by changes in $N^2$, not by changes in Δ$h$.

\[
\Delta KE \approx \frac{1}{2} \left( \frac{N}{g} \right)^2 |\Delta h|^2 - \Delta KE_o
\]

All: $R^2=0.4$
Fixed Δ$h$: $R^2=0.25$
Fixed $N^2$: $R^2=0.02$
Background

• Conservation of the Bernoulli function $B (=KE+PE+enthalpy)$ following a fluid parcel,

\[ \rho \frac{DB}{Dt} = Q_{rad} + Q_{sen} \approx 0 \text{ on timescales of baroclinic waves} \]

• Atmospheric application here using moist enthalpy $h = c_pT + l_vq$
KEmax events with RH>2/3

• Number of events per winter within 5 X 5 degree boxes

• Number of events with the right Bernoulli at low level at the same time (again within 5X5 degree boxes)
KEmax events with $\text{RH} > \frac{2}{3}$

- Number of events per winter within 5 X 5 degree boxes

- Number of events with the right Bernoulli at low level \textit{two days earlier} (again within 5X5 degree boxes)
Further properties of “moist KE\textsubscript{max} events” at 300hPa (ERAinterim, DJF 1980-2012)

- Stronger low level ascent

\textit{RH}>0.6 \textit{RH}<0.6

- Broader distribution of KE (not shown here).
TD model performance for KEmax with RH>0.6 (DJF, 1980-2012)
TD model performance for KE\text{max} with RH<0.6 (DJF, 1980-2012)
The thermodynamic constraint

- At fixed relative humidity, an increase in Tb leads to a decrease in $|\Delta h|$ ("divergence of the moist adiabats").
- This is more than offset by an increase in $|\Delta h|$ due to increasing $q_b$.
- Net effect is an approximate linear relationship between $|\Delta h|$ and $h_b$

\[
|\Delta h| \approx \frac{\Delta T}{T_b} \frac{ma}{h_b} + cst
\]
These features are readily seen in daily snapshots

Black contours:
$(U^2 + V^2)/2$ at 300hPa

Color:
Specific humidity at 800hPa

Magenta:
Negative $\omega$ at 800hPa
Test of the model: actual scatter plots for KE\textsubscript{max} events with RH>2/3

\[ \Delta P E \approx | \Delta h | + \Delta K E_o \]

- Good agreement considering the minimal GFD information

\[ \Delta K E \approx \frac{1}{2} \left( \frac{N}{g} \right)^2 | \Delta h |^2 - \Delta K E_o \]
Test of the model: actual scatter plots for KE_{max} events with RH>2/3

\[ \Delta PE \approx | \Delta h | + \Delta KE_o \]

- Good agreement considering the minimal GFD information

\[ \Delta KE \approx \frac{1}{2} \left( \frac{N}{g} \right)^2 | \Delta h |^2 - \Delta KE_o \]